Resonant electron firehose instability: Particle-in-cell simulations

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12 May 2003

Abstract

Consider a collisionless, homogeneous plasma in which the electron velocity distribution is a bi-Maxwellian with $T_\perp < T_{||}$, where the subscripts refer to directions relative to the background magnetic field $\mathbf{B}_o$. If this anisotropy is sufficiently large and the electron $\beta_{||}$ is sufficiently greater than one, linear dispersion theory predicts that a cyclotron resonant electron firehose instability is excited at propagation oblique to $\mathbf{B}_o$ with growth rates less than the electron cyclotron frequency $|\Omega_e|$ and zero real frequency. This theory at constant maximum growth rate yields threshold conditions for this growing mode of the form $1 - T_{\perp e}/T_{||e} = S'_e/\beta_{||e}^{\alpha'_e}$, where the two fitting parameters satisfy $1 \lesssim S'_e \lesssim 2$ and $\alpha'_e \lesssim 1.0$ over $2.0 \leq \beta_{||e} \leq 25.0$. The first particle-in-cell computer simulations of the resonant electron firehose instability are described here. These simulations show that enhanced magnetic field fluctuations reach a maximum value of $|\delta B|^2/B_o^2$ which increases with $\beta_{||e}$. These enhanced fields scatter the electrons, reducing their anisotropy approximately to a linear theory threshold condition and yielding a dimensionless scattering rate which increases as $\beta_{||e}$ increases. These results are consistent with the general principle that, for a given plasma species, scattering by enhanced fluctuations from anisotropy-driven electromagnetic instabilities acts to make the velocity distribution more nearly isotropic as the $\beta_{||}$ of that species increases.

I. Introduction

Electron and ion velocity distributions in collisionless space plasmas are often observed to be more isotropic than would be predicted by fluid models of charged particle response to large-scale changes in magnetic and electric fields. For example, simple adiabatic theory predicts that solar wind electrons and ions should develop strong $T_{||} >> T_{\perp}$ anisotropies as they stream outward in the decreasing interplanetary magnetic field (The subscripts represent directions relative to the background magnetic field $\mathbf{B}_o$.). Spacecraft observations have shown that velocity distributions of the hot, collisionless components of both ion and electrons are much less anisotropic than predicted by such theories.

A scenario has been proposed to explain these observations. In this scenario a strong anisotropy on the electrons or an ion component in a finite β, collisionless plasma excites one or more electromagnetic kinetic instabilities. The growing modes lead to enhanced electromagnetic fluctuations; because these waves are resonant with the charged particle species driving the instability, they yield strong scattering of these particles, thereby reducing and imposing an observable constraint on the driving anisotropy. In this framework, we have combined computer simulations and spacecraft observations to argue that the proton cyclotron anisotropy instability constrains the $T_{\perp p}/T_{||p} > 1$ anisotropy [Reference 1 and citations therein] and that the proton resonant firehose instability imposes a bound on $T_{||p}/T_{\perp p}$ when that quantity is greater than unity.[2,3] An important general conclusion of our research has been that the larger the $\beta_{||j}$ ($\equiv 8\pi n_j k_B T_{||j}/B_o^2$ for the jth species or component) the more effective is the scattering and the more stringent is the constraint which is imposed on the anisotropy. There has been less progress in demonstrating the validity of this scenario as applied to the electron velocity distributions of observed space plasmas.

This manuscript describes theoretical and computational studies of electromagnetic plasma instabilities which are driven by $T_{\perp e}/T_{||e} < 1$ (Subscript e denotes electrons). Although instabilities excited by $T_{\perp e}/T_{||e} > 1$ have been frequently simulated,[4−10] only

Messmer[11] has published particle-in-cell simulations of modes driven by the opposite electron anisotropy.

For both the linear theory and particle-in-cell simulations described here, we assume a homogeneous, magnetized, collisionless plasma; we also assume the electrons are represented by a single component bi-Maxwellian velocity distribution. We admit that this is an idealized distribution, as electrons observed in the magnetosphere and solar wind usually exhibit two or more components and in the solar wind bear a heat flux. Nevertheless, it is appropriate to seek an understanding of the consequences of instabilities which arise in this model to establish a baseline before pursuing studies of more realistic but more complex electron velocity distributions.

As we explain in Section II below, linear dispersion theory for the single bi-Maxwellian electron distribution model predicts that two, distinct electron firehose instabilities can be excited for sufficiently large values of $1-T_{\perp e}/T_{\|e}$ and $\beta_{\|e}$. Linear dispersion theory further predicts that the threshold condition for each of these instabilities can be written as

$$1-\frac{T_{\perp e}}{T_{\|e}}=\frac{S_e'}{\beta_{\|e}^{\alpha_e'}} \tag{1}$$

where the two primed quantities are fitting parameters with $1 \lesssim S_e' \lesssim 2$ and $\alpha_e' \lesssim 1.0$. Particle-in-cell simulations at $\mathbf{k}\times\mathbf{B}_o=0$ have shown that wave-particle interactions by the enhanced fluctuations reduce this anisotropy and thereby stabilize this growing mode.[11] However, simulations have not yet demonstrated whether Equation (1) represents an observable constraint. Paesold *et al.*[12] used linear theory and test-particle computations to argue that the electron firehose instability may play a role in the acceleration and enrichment of ^3He during impulsive solar flares.

For both the linear theory and the simulations described here, we consider two species: ions (denoted by subscript i) and electrons. For the jth species we define $\beta_{\|j} \equiv 8\pi n_j k_B T_{\|j}/B_o^2$; the plasma frequency, $\omega_j \equiv \sqrt{4\pi n_j e_j^2/m_j}$; the cyclotron frequency, $\Omega_j \equiv e_j B_o/m_j c$; and the thermal speed, $v_j \equiv \sqrt{k_B T_{\|j}/m_j}$. The Alfvén speed is $v_A \equiv B_o/\sqrt{4\pi n_i m_i}$. The complex frequency is $\omega = \omega_r + i\gamma$, the Landau resonance factor of the jth species is $\zeta_j \equiv \omega/\sqrt{2}|k_\parallel|v_j$, and the cyclotron resonance factors of the jth species are $\zeta_j^\pm \equiv (\omega \pm \Omega_j)/\sqrt{2}|k_\parallel|v_j$. The choice of coordinate system is such that both $\mathbf{B}_o$ and the wavevector $\mathbf{k}$ lie in the x-z plane. We define θ as the angle between $\mathbf{k}$ and $\mathbf{B}_o$, so that $\hat{\mathbf{k}} \cdot \hat{\mathbf{B}}_o = \cos(\theta)$. Subscript m denotes a quantity corresponding to the maximum growth rate γ_m/Ω_i; thus k_m and θ_m correspond to the wavevector which yields the largest value of γ for a given set of dimensionless plasma parameters.

II. Linear Theory

This section describes properties of two electromagnetic instabilities driven by $T_{\perp e}/T_{\parallel e} < 1$ as derived from the linear Vlasov dispersion equation in a homogeneous, collisionless, magnetized plasma. We assumed that the ions are represented as an isotropic Maxwellian, that the electrons may be represented by a single anisotropic bi-Maxwellian velocity distribution, that the average relative drift between the electrons and ions is zero, and that charge neutrality $n_e = n_i$ holds. In this case the dispersion equation is derived and discussed in Chapter 7 of Reference 13. We assumed the following dimensionless parameters for solutions of this equation: $m_i/m_e = 1836$, $v_A/c = 1.0 \times 10^{-4}$, $T_{\parallel e}/T_{\parallel i} = 1$ and, to isolate the consequences of the electron anisotropy, $T_{\perp i}/T_{\parallel i} = 1$.

If $(1 - T_{\perp e}/T_{\parallel e})$ and $\beta_{\parallel e}$ are both sufficiently large, an electron anisotropy instability arises at $\mathbf{k} \times \mathbf{B}_o = 0$[14–16] with $\omega_r \neq 0$ and relatively weak growth rates ($\gamma_m/\Omega_i < 1$). Because both species velocity distributions are symmetric about $v_\parallel = 0$, both right- and left-propagating instabilities arise with the same growth rates and the same left-hand circular polarization. The electrons are nonresonant ($|\zeta_e^\pm| \sim 3$), whereas the ions satisfy $|\zeta_i^\pm| \simeq 0$.[17] We call this the "nonresonant electron firehose instability." We calculated numerical solutions of the linear Vlasov dispersion equation at $\mathbf{k} \times \mathbf{B}_o = 0$ for two different values of the maximum growth rate; we then used a numerical least-squares fit to the results to obtain an expression for the threshold condition. Over $2.0 \leq \beta_{\parallel e} \leq 50.0$ we found that the threshold condition satisfies Equation (1) with

$$S'_e = 1.70 \qquad \text{and} \qquad \alpha'_e = 0.99 \qquad \text{at} \qquad \gamma_m/\Omega_i = 0.01$$

$$S'_e = 1.62 \qquad \text{and} \qquad \alpha'_e = 0.94 \qquad \text{at} \qquad \gamma_m/\Omega_i = 0.10$$

The latter case is plotted as the solid dots and the solid line in Figure 1.

Again, if $(1 - T_{\perp e}/T_{||e})$ and $\beta_{||e}$ are both sufficiently large, a different instability arises at propagation oblique to the magnetic field with $\omega_r = 0$ and $\Omega_i < \gamma < |\Omega_e|$.[17,18] Figure 2 illustrates the growth rate of this mode maximized with respect to wavenumber as a function of θ for both the nonresonant instability and this second mode which we call the "resonant electron firehose instability." We chose this name because, by comparison with the $\mathbf{k} \times \mathbf{B}_o = 0$ mode, at maximum growth the electrons have a relatively strong cyclotron resonance with $|\zeta_e^{\pm}| \sim 2$. At γ_m the ion cyclotron resonance factor satisfies $|\zeta_i^{\pm}| << 1$ and the Landau resonance of course corresponds to $\mathrm{Re}(\zeta_e) = \mathrm{Re}(\zeta_i) = 0$.

Figure 2 (compare with the figures of References 17 and 18) shows that the maximum growth rate of the nonresonant mode remains smaller than Ω_i at oblique propagation, but that the maximum growth rate of the resonant electron mode becomes much larger than the ion cyclotron frequency as θ increases.

This suggests that the resonant electron instability has a lower threshold than the nonresonant mode,[18] and a detailed calculation illustrated in Figure 1 confirms this. We found that the resonant electron instability threshold also satisfies Equation (1) but with fitting parameters over $2.0 \leq \beta_{||e} \leq 25.0$ as described in Table I. The results from this table are based on $m_i/m_e = 1836$, but we have also carried out a similar set of threshold calculations for $m_i/m_e = 100$ for use in the simulations described in the next section and have found essentially the same results. Thus we conclude that the linear theory properties of the resonant firehose instability are essentially independent of m_i/m_e at sufficiently large mass ratios.

Figure 1 implies that, unless some physical property of the system restricts mode propagation to nearly parallel or antiparallel to $\mathbf{B}_o$, the resonant firehose instability should be the dominant mode. Therefore our simulations described in the following section address this instability.

The zero real frequency of the resonant electron firehose mode invites comparison with the electron mirror instability which also has $\omega_r = 0$ but is driven by $T_{\perp e}/T_{\| e} > 1$. Both modes have maximum growth rate at propagation oblique to $\mathbf{B}_o$ with, at sufficiently great anisotropies, $\Omega_i << \gamma_m < |\Omega_e|$. However, electrons do not have a cyclotron resonance with the electron mirror instability, as $|\zeta_e^{\pm}| \gtrsim 5$ for this mode. In addition the electron mirror instability is, like its ion-driven counterpart, predominantly compressive; that is, $|\delta B_{\|}|^2 >> |\delta B_{\perp}|^2$, whereas the resonant electron firehose instability fluctuations *are predominantly transverse to* $\mathbf{B}_o$*:* $|\delta B_{\perp}|^2 >> |\delta B_{\|}|^2$. Because the latter instability is both cyclotron resonant with electrons and predominantly transverse, it is likely that pitch-angle scattering is the primary mechanism for anisotropy reduction by enhanced fluctuations from this mode.

III. Particle-in-cell simulations

The particle-in-cell simulation code used here is described in Reference 10. The code is a two-and-one-half dimensional; that is, all three velocity dimensions are computed, but spatial variations are permitted only in the x-z plane. For all simulations reported here, the following initial parameters were used: $m_i/m_e = 100$, $v_A/c = 0.00707$, $T_{\perp i}/T_{\| i} = 1$, and $T_{\| e}/T_{\| i} = 1$. We used 400 electrons and 400 ions per cell in each run, and a grid spacing of $\Delta x = \Delta z = \lambda_D$, the Debye wavelength. Because the resonant electron firehose instability has the far larger growth rate in the model we assumed, our simulation coordinates were chosen to capture the strongly oblique character of this unstable mode. Thus we chose the x-direction to be parallel to the wavevector at maximum growth rate, and the simulation length in this direction, L_x, to be equal to four times the wavelength of the fastest growing mode. Because of computational limitations, we chose only three cells in the z-direction, so that, in effect, our simulations represent spatial variations in only one dimension.

We first carried out a representative simulation of the resonant electron firehose instability, Run F-257 with initial parameters as stated in Table II. Selected results from this computation are illustrated in Figure 3 which shows that the fluctuating magnetic fields grow at a rate close to that predicted by linear theory, and both the electron anisotropy and the $\beta_{\|e}$ are reduced at a much slower rate. The ion response is uninteresting, with no significant change either in the overall ion temperature or in $T_{\perp i}/T_{\|i}$ for the duration of the simulation; thus, we do not illustrate any ion quantities here. Throughout the growth, saturation, and subsequent decay of the fields, $|\delta B_x|^2 << |\delta B_z|^2 << |\delta B_y|^2$; that is, the primary contribution to $|\delta B|^2/B_o^2$ illustrated in Figure 3 is due to the out-of-the-simulation-plane component of the fluctuating magnetic field. Although they are not illustrated here, we have also plotted contours of δB_y at several times during the simulation. These plots clearly demonstrate that the fluctuating fields during the growth phase are due to a zero frequency mode, consistent with the predictions of linear theory. Fourier plots of individual mode amplitudes as functions of time demonstrate the expected quasilinear response; that is, the $n = 4$ mode usually has the fastest initial growth and the earliest saturation, while the $n = 3$ and $n = 2$ modes display successively slower growth and later saturation times. The two peaks in Figure 3a correspond to saturation of the $n = 4$ and $n = 3$ modes, respectively.

We examined the reduced electron velocity distributions at several times during the representative simulation. We find that all three of the $f_e(v_j)$ remain approximately bi-Maxwellian throughout the computation, even as the anisotropy is substantially reduced. Although our limited box size allows only a few modes to grow to large amplitude, the fast growth rate of the waves leads to relatively broad cyclotron and Landau resonances. This permits electrons with a broad range of $v_{\|}$ to be scattered and allows the bi-Maxwellian character of the velocity distribution to be maintained.

We also examined the magnetic fluctuation distributions for δB_y and δB_z during Run

F-257. The magnetic fluctuations were taken as spatial differences; that is, $\delta B_j(\delta x, t) \equiv B_j(\mathbf{x}+\delta x, t) - B_j(\mathbf{x}, t)$ and the distributions were computed by binning the values of δB_j over all the cells of the simulation. We examined magnetic fluctuation distributions at seven different spatial separations: $\delta x\ \omega_e/c = 10$, 20, 30, 40, 50, 60 and 70, and four times: $|\Omega_e|t = 10$, 20, 30 and 40. The $f(\delta B_z)$ were Gaussian-like for most separations and most times, with the greatest departures from a Gaussian at $|\Omega_e|t = 20$ and $\delta x\ \omega_e/c = 40$ and 50. In contrast, $f(\delta B_y)$ exhibited significant departures from Gaussian shapes as early as $|\Omega_e|t = 10$ and showed the strongest non-Gaussian profiles at $|\Omega_e|t = 30$, the approximate time of saturation. Figure 4 illustrates the $f(\delta B_y)$ at $\delta x\ \omega_e/c = 30$ and several times during Run F-257. The bimodal distribution at $|\Omega_e|t = 30$ corresponds to the dominance of a single mode in the system at this time which of course is the opposite condition from that of turbulence or many waves with random relative phases which typically leads to a Gaussian-like distribution.

We next carried out an ensemble of simulations of the resonant electron firehose instability. We chose four initial values of $\beta_{||e}$, used linear theory to determine the corresponding electron anisotropies which yielded initial $\gamma_m/|\Omega_e| = 0.20$, and chose L_x values to match four times the wavelength of the fastest growing mode in each case. We then ran the simulations to $|\Omega_e|t = 100$ using the initial parameters described in Table II. Figure 5 provides a graphical summary, plotting the electron anisotropy versus $\beta_{||e}$ at several times during each simulation of the ensemble. In all four cases, scattering by the enhanced fluctuations reduced the electron anisotropy to the approximate instability threshold condition of $\gamma_m/|\Omega_e| = 0.10$ at saturation. After saturation the scattering rate became much weaker, but there was a continuing trend toward smaller anisotropies and, at the larger values of $\beta_{||e}$, toward conditions of still weaker growth. Simulations in larger systems would, we believe, show the post-saturation excitation of longer wavelength modes with still smaller growth rates; it is likely that the consequent scattering would further reduce the electron anisotropies to threshold conditions of still weaker $\gamma_m/|\Omega_e|$.

Table III summarizes additional results from the ensemble of simulations, providing the maximum values of the fluctuating magnetic field energy density and the electron anisotropy scattering rate ν_e for each computation. For a fixed initial value of $\gamma_m/|\Omega_e|$ the maximum $|\delta B|^2/B_o^2$ increases with increasing $\beta_{\|e}$; this is the same type of scaling as was obtained for the whistler anisotropy instability driven by $T_{\perp e}/T_{\|e} > 1$.[10] To estimate the maximum electron scattering rate we assumed that the anisotropy satisfies

$$1 - \frac{T_{\perp e}}{T_{\|e}} = A_o \exp(-\nu_e t) \tag{2}$$

and then fit Equation (2) to a computed anisotropy versus time plot over $15 \leq |\Omega_e|t \leq 25$ which encompasses the times of fastest scattering for each run. Table III reports the resulting values of $\nu_e/|\Omega_e|$ as the maximum electron scattering rate for a given set of initial parameters. Here $\nu_e/|\Omega_e|$ increases as $\beta_{\|e}$ becomes larger; this is opposite to the $\beta_{\|e}$ dependence found by Nishimura *et al.*[10] for the scattering rate due to whistler anisotropy instability.

IV. Conclusions

We used linear Vlasov theory to compare the properties of two growing modes excited by $T_{\perp e}/T_{\|e} < 1$: the nonresonant electron firehose instability with $\omega_r \neq 0$ and $\gamma_m < \Omega_i$ at $\mathbf{k} \times \mathbf{B}_o = 0$, and the resonant electron instability with $\omega_r = 0$ and $\gamma_m < |\Omega_e|$ at propagation only oblique to $\mathbf{B}_o$. For all parameters we considered, the latter mode has the far larger growth rate, and the lower anisotropy threshold with the form of Equation (1). Therefore, we carried out a series of particle-in-cell simulations addressing the latter mode. The results described here are, to the best of our knowledge, the first report of simulations of the resonant electron firehose instability.

We draw several conclusions from our simulations. (1) Enhanced field fluctuations excited by the resonant electron firehose instability increase as $\beta_{\|e}$ increases above unity. (2) These enhanced fluctuations scatter the electrons, thereby reducing their anisotropy; this scattering preserves the bi-Maxwellian character of the electron velocity distribution. (3) For a fixed maximum growth rate the maximum value of the dimensionless anisotropy scattering rate increases as $\beta_{\|e}$ increases. (4) This scattering reduces the electron anisotropy to an instability threshold condition of weaker growth. The implication of this last conclusion is that the resonant electron firehose instability has the potential to impose a β-dependent upper bound on the electron anisotropy, and that it would be a useful exercise to seek such a constraint in observations from the solar wind and magnetosphere.

We have also examined magnetic fluctuation distributions due to the resonant electron firehose instability. We find that near the maximum amplitude of the fluctuating fields, the $f(\delta B_y)$ are strongly non-Gaussian due to the predominance of a few, coherent modes in the system. We suggest that relatively short-wavelength electromagnetic instabilities driven by electron and ion anisotropies may contribute to recent solar wind observations that magnetic fluctuation distributions exhibit an increasing departure from the Gaussian as frequencies (and wavenumbers) increase.[19]

Our simulations were carried out in a collisionless, homogeneous plasma model. However, because the resonant instability wavelength at maximum growth rate scales approximately as the thermal electron gyroradius, relatively long wavelength inhomogeneities should not inhibit the growth of this mode.

In our opinion the major limitation on the application of this instability to space plasmas is not stabilization by inhomogeneities, but rather the lack of observations of the idealized, single bi-Maxwellian electron distribution used in our theory and simulations. For example, electron velocity distributions observed in the solar wind are often characterized in terms of three distinct components: a cool, relatively isotropic core, a tenuous, hot, strongly anisotropic strahl, and a still more tenuous, hot, relatively isotropic halo.[20] Although the core and halo typically can be approximated by velocity distributions such as used here, the strahl is a unidirectional, highly focused distribution which may not be subject to a bi-Maxwellian representation. A natural extension of this work would be to examine the linear theory and simulation properties of firehose-like and heat-flux instabilities driven by such multi-component velocity distributions.

Nevertheless, we believe that the computations described here provide a general principle for comparison against future simulations of instabilities and future observations. This principle applies to instabilities driven by both electron and ion anisotropies, and to both $T_\perp/T_\| > 1$ and $T_\perp/T_\| < 1$. This principle may be stated as: For a given plasma species, scattering by enhanced fluctuations from anisotropy-driven electromagnetic instabilities acts to make the velocity distribution more nearly isotropic as the $\beta_\|$ of that species increases. This is consistent with some solar wind electron observations. These include the $\beta_{\|e}$-dependent upper bound on $T_\|/T_\perp$ evident in the analysis of halo electrons measured from the Ulysses spacecraft[21] and measurements from the Wind spacecraft showing that suprathermal electrons become more nearly isotropic as the electron β increases.[22]

Acknowledgments. We acknowledge useful conversations with Bill Daughton and Hui Li. This work was performed at Los Alamos National Laboratory under the auspices of the U.S. Department of Energy (DOE) and was supported by the DOE Office of Basic Energy Sciences, Division of Engineering and Geosciences, the Laboratory Directed Research and Development Program at Los Alamos, and the Sun-Earth Connection Guest Investigator Program of the National Aeronautics and Space Administration.

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Table I. Linear Theory Results: Fitting Parameters for Equation (1) at Thresholds for Resonant Electron Firehose Instability

$\gamma_m/\|\Omega_e\|$	S'_e	α'_e
0.001	1.29	0.97
0.010	1.23	0.88
0.025	1.22	0.79
0.050	1.26	0.71
0.10	1.32	0.61
0.20	1.36	0.47

Table II. Initial Parameters for the Ensemble of Simulations

Run	$\beta_{\parallel e}$	$T_{\perp e}/T_{\parallel e}$	$\gamma_m/\lvert\Omega_e\rvert$	$k_m c/\omega_e$	θ_m (degrees)	$L_x \omega_e/c$
F-252	2.5	0.10	0.20	1.32	77.0	19.1
F-255	5.1	0.34	0.21	0.79	75.0	31.8
F-257	7.7	0.46	0.21	0.65	75.0	38.9
F-250	9.9	0.55	0.19	0.57	75.5	43.6

Table III. Summary Results from the Ensemble of Simulations

Run	Initial $\beta_{\parallel e}$	Initial $T_{\perp e}/T_{\parallel e}$	$\lvert\Omega_e\rvert t$ of maximum fields	Maximum $\lvert\delta B\rvert^2/B_o^2$	Maximum $\nu_e/\lvert\Omega_e\rvert$
F-252	2.5	0.10	25	0.016	0.003
F-255	5.1	0.34	27	0.048	0.011
F-257	7.7	0.46	30	0.067	0.015
F-250	9.9	0.55	30	0.077	0.016

Figure Captions

Figure 1. Linear theory results: Electron anisotropies at thresholds of two instabilities as functions of parallel β_e for two different values of the maximum instability growth rate. In each case the discrete symbols represent linear theory results; the lines are least-squares fits to these points. The solid line corresponds to the nonresonant electron firehose instability at $\gamma_m/\Omega_i = 0.10$ and the dashed line represents the $\gamma_m/|\Omega_e| = 0.001$ threshold of the resonant electron firehose instability. Parameters are as stated in Section II.

Figure 2. Linear theory results: The growth rates maximized over wavenumber and the corresponding wavenumbers as functions of the direction of propagation for both the nonresonant electron firehose instability with $\omega_r \neq 0$ and the resonant electron firehose instability with $\omega_r = 0$. Parameters are as stated in Section II with, in addition, $\beta_{||e} = 5.0$ and $T_{\perp e}/T_{||e} = 0.60$.

Figure 3. Results from the simulation Run F-257: (a) the fluctuating magnetic field energy density, (b) the electron anisotropy and (c) the electron parallel β as functions of time. The dashed line in panel (a) represents the fluctuating magnetic field energy density growing at the initial maximum growth rate of $\gamma_m/|\Omega_e| = 0.20$.

Figure 4. Results from the simulation Run F-257: The magnetic fluctuation distributions of δB_y at four times as labeled.

Figure 5. Results from the ensemble of simulations. For each run with initial conditions as described in Table II, the electron anisotropy is plotted as a function of $\beta_{||e}$. Initial conditions are plotted as crosses, conditions at saturation of the fluctuating magnetic fields are plotted as open circles, and conditions at $|\Omega_e|t = 50$, 75 and 100 are plotted as solid dots. The dashed line represents the linear theory threshold condition for the resonant electron firehose instability at $\gamma_m/|\Omega_e| = 0.10$.